

CHARACTERIZATION OF EVOLVING PLASTIC ANISOTROPY AND ASYMMETRY OF A RARE-EARTH MAGNESIUM ALLOY SHEET BY MEANS OF A NON-ASSOCIATED FLOW RULE

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Abstract. The superior ductility of rare-earth magnesium alloys over conventional magnesium sheets makes them promising candidates for light-weight structural alloys. However, these alloys possess severe evolving anisotropy and tension-compression asymmetry as a result of activation of different deformation mechanisms (slip or twinning) that is extremely challenging to model numerically. In this study, the constitutive plastic behaviour of a rare-earth magnesium alloy sheet, ZEK100 (O-temper), was considered at room temperature, under quasi-static conditions. A CPB06 yield criterion for hcp materials was employed along with a non-associated flow rule where the yield function and plastic potential were calibrated at different plastic deformation levels to account for evolving anisotropy in proportional loading. The constitutive model was implemented as a user material subroutine (UMAT) into the commercial finite element package, LS-DYNA, along with an interpolation technique to consider the evolving anisotropy of the material. Finally, predictions of the model were compared with the experimental results in terms of flow stresses and plastic flow directions under various proportional loading conditions and along different test directions. It was shown that the predictions of the model were in good agreement with experimental data.

1 INTRODUCTION

Most commercial alloys used in sheet metal forming and vehicle crashworthiness applications exhibit some degree of orientation-dependent plastic response, and depending on the severity of plastic anisotropy, isotropic yield functions might not be suitable candidates for modelling the behaviour of the materials. To overcome this issue, a large number of anisotropic yield functions have been proposed in the literature with the largest contributions from the Barlat family of yield criteria [1,2] in which linear transformations are applied on the stress tensor to account for anisotropy. However, these models were intended for bcc and fcc cubic materials with slip-dominated deformation mechanisms while magnesium alloys have

an hcp crystal structure. In hcp materials, plastic deformations occur by slip and twinning mechanisms and due to the direction sensitivity of twinning mechanisms, strong tension-compression asymmetry is observed in yield loci [3]. In order to account for anisotropy and strength differential effects, Cazacu *et al.* [4] proposed a new yield function (known as the CPB06 yield criterion) based on the linear transformation approach. Later, Plunkett *et al.* [5] showed that the predictions of the CPB06 yield function can be improved if more than one linear transformation is applied on the stress tensor.

Due to texture evolutions during plastic deformation, the shape of the yield surface of magnesium alloys does not remain constant, thus, isotropic hardening models cannot capture the material behaviour accurately. To consider evolving anisotropy, Plunkett *et al.* [5] proposed a piece-wise linear interpolation of the CPB06 yield functions calibrated at different levels of plastic deformation. A similar approach was also adopted by Ghaffari Tari *et al.* [6] for AZ31B magnesium. These models were based on assumption of the associated flow rule (AFR) in which the yield function is also the plastic potential for calculating the plastic strain increments (normality rule). The classical work of Bishop and Hill [7] demonstrated that the AFR holds for metals based on a crystal plasticity model. However, in the last decades, the assumption of the AFR has been challenged by the non-associated flow rule (non-AFR) in which the plastic potential is independent from the yield function [8,9]. Considering materials with severe anisotropy such as magnesium alloys, the non-AFR provides more degrees of flexibility for calibrating yield stresses and r-values. Furthermore, models with the non-AFR enable the possibility that plastic potential and yield functions evolve independently, a feature that is not possible in models based on the AFR. These characteristics make the non-AFR attractive for materials such as ZEK100 with strong evolving anisotropic behaviour.

The objectives of the present work are to investigate and model the anisotropic plastic response of a rare-earth magnesium alloy sheet (ZEK100-O) under quasi-static conditions. To this end, the non-AFR is employed by calibrating yield function and plastic potential at different plastic deformation levels using the CPB06 formulation with two linear stress transformations. The model was implemented into the commercial finite element package, LS-DYNA, along with an interpolation technique to consider the evolving anisotropy of the material. The finite element model comprises a single 3-D element that is subjected to various stress states in different test orientations with respect to the rolling direction. The outcomes of each single-element simulations in terms of flow stresses and r-values are compared to the experimental results of Abedini *et al.* [3] to assess the predictive capabilities of the model.

2 MATERIAL AND EXPERIMENTAL RESULTS

A rare-earth magnesium alloy ZEK100-O rolled sheet with a nominal thickness of 1.55 mm was used in the present study. An extensive experimental investigation into the anisotropy of this same lot of material was performed by Abedini *et al.* [3] and this test data will be utilized in the present paper to develop the constitutive model. Experimental tests in [3] were performed at room temperature, under a quasi-static strain rate of 0.001 s^{-1} . Constitutive plastic behaviour of the material is shown in Figure 1 in terms of stress response and r-values, and it can be seen that the material exhibits significant anisotropy that evolves with deformation. Furthermore, it is apparent from Figure 1(a-c) that ZEK100-O exhibits a

tension-compression asymmetric response which is due to the twinning mechanisms that are more dominant under compression mode [10]. Moreover, the behaviour of the material under shear state (Figure 1e) shows anisotropic trends. It is recently shown in [3,11] that in addition to the asymmetric response in the first and third quadrants of yield loci (tensile and compressive regions), twinning mechanisms activated under shear deformation leads to an additional asymmetry in the second and fourth quadrants for magnesium alloys. The experimental results shown in Figure 1 show the challenging nature of ZEK100-O in terms of modelling and characterization and highlight the need for accurate constitutive plasticity models that are able to capture the evolving anisotropy of the material.

3 PLASTICITY MODEL

In order to consider the anisotropic and asymmetric response of ZEK100-O, the CPB06 phenomenological yield criterion proposed by Cazacu *et al.* [4] was adopted in the present study. In analogy to Plunkett *et al.* [5], two stress transformations were performed to increase the flexibility of the model (denoted as CPB06ex2). The yield function, Φ , is defined as:

$$\Phi = (|\Sigma_1| - k \Sigma_1)^a + (|\Sigma_2| - k \Sigma_2)^a + (|\Sigma_3| - k \Sigma_3)^a + (|\Sigma'_1| - k' \Sigma'_1)^a + (|\Sigma'_2| - k' \Sigma'_2)^a + (|\Sigma'_3| - k' \Sigma'_3)^a \quad (1)$$

where k and k' are material parameters that account for strength differential effects, and a is the exponent of the yield function. Also Σ_{1-3} and Σ'_{1-3} are the principal values of the transformed stress deviators Σ_{ij} and Σ'_{ij} written as:

$$\Sigma_{ij} = C_{ijkl} : S_{kl} \quad \text{and} \quad \Sigma'_{ij} = C'_{ijkl} : S_{kl} \quad (2,3)$$

where S_{kl} and C_{ijkl} are the deviatoric stress tensor and the fourth-order transformation tensor, respectively. The non-associated flow rule was employed in the present study with the same functional form as Eq. (1) to define the plastic potential, Ψ , to which the plastic strain components are normal and their magnitudes are governed by:

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial \Psi}{\partial \sigma_{kl}} \quad (4)$$

where $d\varepsilon_{ij}^p$ is the incremental plastic strain tensor, and $d\lambda$ is the plastic multiplier. The values of coefficients of the transformation tensors and strength differential parameters can be determined from an optimization approach to minimize the errors between the experimental data and the values predicted by the yield function and plastic potential. In the present study, the genetic algorithm (GA) which is a global optimizer available in Matlab[®] was used to calculate these parameters.

As shown in Figure 1, ZEK100-O exhibits an evolving anisotropic behaviour and therefore, the yield function and plastic potential should be calibrated at different plastic work levels, w_p . To capture the evolution, a piece-wise linear interpolation technique can be used:

$$\Phi = \xi(w^p) \Phi_1 + [1 - \xi(w^p)] \Phi_2 \quad \text{and} \quad \Psi = \xi(w^p) \Psi_1 + [1 - \xi(w^p)] \Psi_2 \quad (5,6)$$

in which Φ_1 (Ψ_1) and Φ_2 (Ψ_2) are the yield function (plastic potential) at the deformation levels associated with the plastic works of w_n^p and w_{n+1}^p , respectively, and $\xi(w^p)$ is calculated by:

$$\xi(w^p) = (w_{n+1}^p - w^p) / (w_{n+1}^p - w_n^p) \quad (7)$$

In terms of the hardening behaviour of the material, a Hockett-Sherby function (Eq. 8) was calibrated to the uniaxial tensile data in the RD (reference direction) up to maximum strain limited by the onset of necking:

$$\bar{\sigma} = 205.59 + 126.60 [1 - \exp(-1.91(\varepsilon^p)^{0.46})] \quad (8)$$

where $\bar{\sigma}$ is the flow stress and ε^p is the equivalent plastic strain.

The constitutive model described above was implemented as a user material subroutine (UMAT) into the explicit commercial finite element software package, LS-DYNA, using a standard return-mapping algorithm (convex cutting algorithm in Ortiz and Simo [11]). The reader is referred to [8] for the proof of the uniqueness of the stress and strain states as well as the proof of the stability in the non-AFR.

4 RESULTS AND DISCUSSION

To capture the evolving anisotropy of ZEK100-O, nine different plastic work levels for a plastic work range of 2.24 MPa to 22.46 MPa (associated with 1% to 9% equivalent plastic strains in uniaxial tension along the RD) were selected and their corresponding experimental data were used to calibrate yield functions and plastic potentials. Due to brevity, only three levels of plastic deformations (plastic work levels of 2.24 MPa, 14.61 MPa, and 22.46 MPa with anisotropy coefficients presented in Tables 1 and 2) were chosen and their associated yield functions and plastic potentials are shown in Figure 2. It can be seen from Figure 2 that the CPB06ex2 yield function and plastic potential fit the measured data with good accuracy. For the smallest plastic work level of 2.24 MPa which is close to the initial yielding of the material, it can be seen from Figure 2(a) that the material has a clear tension-compression asymmetry with the tension region having larger yield stresses than the compression region. The tension-compression asymmetry of yield loci reduces with deformation due to the high hardening rate in compression offsetting its initially lower yield strength with respect to tension as shown in Figure 2(b) for the plastic work level of 14.61 MPa. At a plastic work level of 22.46 MPa (Figure 2c) the yield stresses in compression have grown larger than in tension which is opposite to that observed for the material at the onset of yielding.

In order to further evaluate the accuracy of the model, a single 3-D brick element under different loading conditions was used to simulate the stress-strain curves and evolution of r-values of the material. Figure 1 compares the predictions of the model with experimental results where it can be seen that the model with the non-AFR is capable of capturing the experimental trends with good accuracy. It should be noted that the FE stress data may exhibit

piece-wise discontinuities due to the inherent nature of the piece-wise linear interpolation approach. This issue may be resolved by better numerical fits (*i.e.* yield functions and plastic potentials with higher flexibility) and increasing the number of calibration levels. Further validations of the model by full-scale FE simulations of structural experiments will be considered in future work.

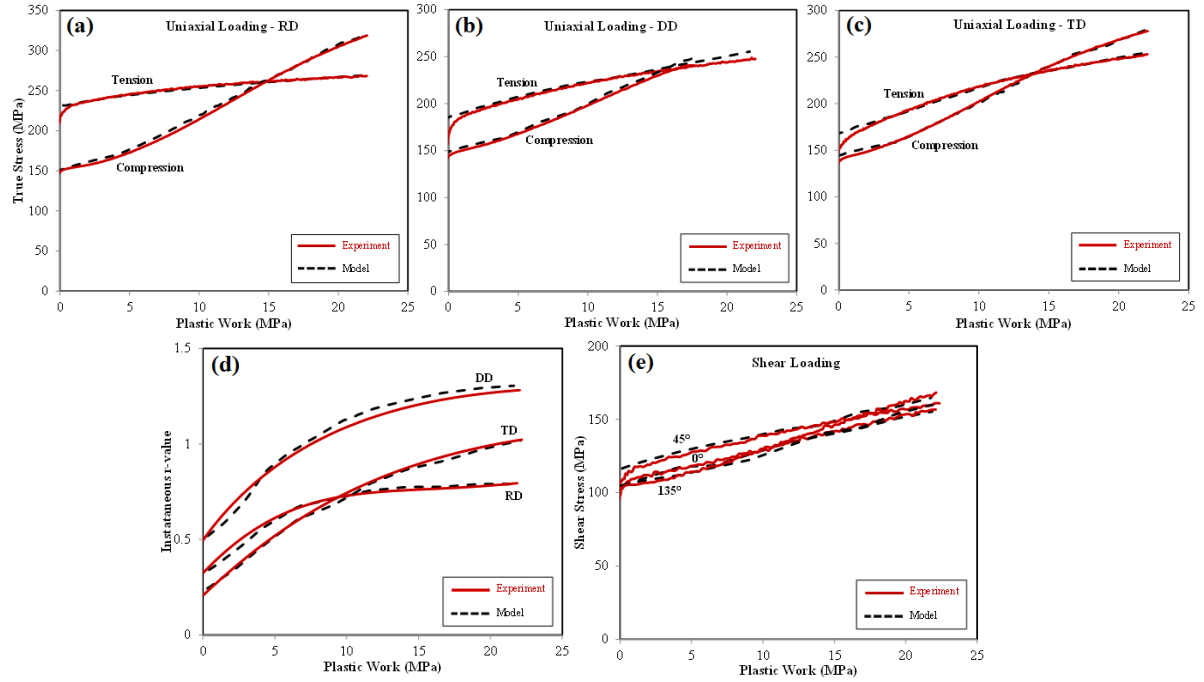


Figure 1: Constitutive plastic behaviour of ZEK100-O in quasi-static conditions. (a-c) show the uniaxial tensile response of the material (d) shows the r-value evolutions, and (e) shows the shear response. All the figures are plotted with respect to the plastic work.

Table 1: Coefficients of the CPB06ex2 yield function.

| w_p (MPa) | C_{11} | C_{12} | C_{13} | C_{22} | C_{23} | C_{33} | C_{66} | k | C'_{11} | C'_{12} | C'_{13} | C'_{22} | C'_{23} | C'_{33} | C'_{66} | k' | a |
|----------------|----------|----------|----------|----------|----------|----------|----------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-------|-----|
| 2.24 | 1.00 | 1.16 | -1.34 | -0.73 | 1.25 | -1.34 | -2.43 | -0.03 | 1.00 | 2.67 | -0.64 | 0.66 | -0.34 | -1.78 | 1.79 | 0.21 | 8.0 |
| 14.61 | 1.00 | 0.91 | -2.06 | 3.70 | -1.05 | -2.93 | 2.80 | -0.01 | 1.00 | -2.20 | -1.70 | -0.11 | 2.32 | -0.04 | 3.61 | -0.02 | 8.0 |
| 22.46 | 1.00 | 1.92 | 1.92 | 3.11 | 0.67 | -0.41 | 1.61 | -0.13 | 1.00 | -1.20 | 0.51 | -0.20 | 1.01 | -0.76 | 1.97 | 0.01 | 8.0 |

Table 2: Coefficients of the CPB06ex2 plastic potential.

| w_p (MPa) | C_{11} | C_{12} | C_{13} | C_{22} | C_{23} | C_{33} | C_{66} | k | C'_{11} | C'_{12} | C'_{13} | C'_{22} | C'_{23} | C'_{33} | C'_{66} | k' | a |
|----------------|----------|----------|----------|----------|----------|----------|----------|-----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------|-----|
| 2.24 | 1.00 | -1.02 | -0.70 | -1.14 | -1.02 | 1.68 | 2.36 | 0.0 | 1.00 | -0.97 | 2.05 | 0.85 | 1.89 | 2.06 | 2.35 | 0.0 | 8.0 |
| 14.61 | 1.00 | 1.53 | 1.35 | 0.72 | 0.27 | 1.32 | 0.32 | 0.0 | 1.00 | 2.49 | 0.93 | 1.02 | 0.92 | 2.48 | 1.82 | 0.0 | 8.0 |
| 22.46 | 1.00 | -0.33 | 1.57 | -1.57 | 0.76 | -0.50 | 3.01 | 0.0 | 1.00 | -0.47 | -1.87 | -1.80 | 0.97 | -0.53 | 3.05 | 0.0 | 8.0 |

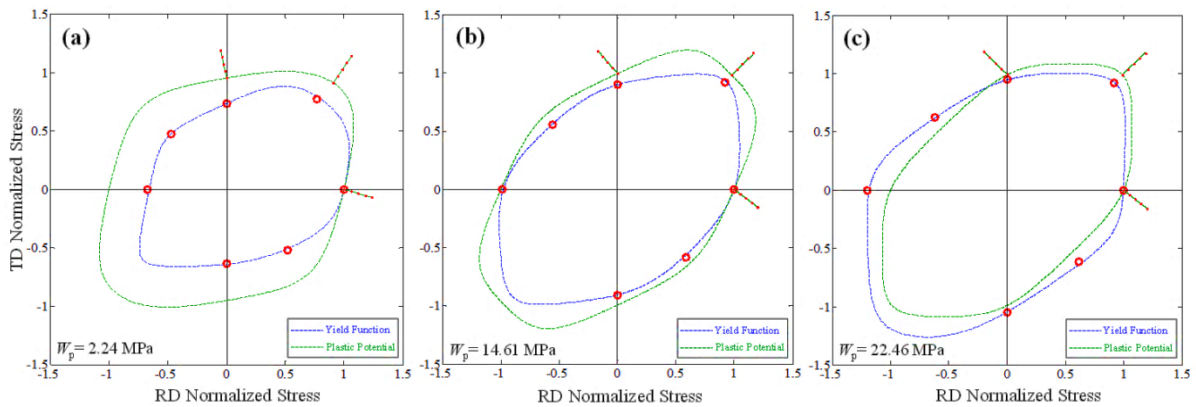


Figure 2: Yield function and plastic potential of ZEK100-O depicted at three different plastic work levels of (a) 2.24 MPa, (b) 14.61 MPa, and (c) 22.46 MPa. Red symbols show the experimental data. The lines normal to plastic potentials show the direction of plastic flow.

5 CONCLUSIONS

The room temperature constitutive plastic behaviour of a rare-earth magnesium alloy sheet, ZEK100-O, was studied under different stress states. It was demonstrated that the material exhibits severe plastic anisotropy that evolves with deformation. It was shown that adopting the non-AFR along with an evolving CPB06 formulation with two linear stress transformations resulted in good agreements between FE predictions and experimental data. The strategy adopted to consider evolution was piece-wise linear interpolations between yield functions and plastic potentials that were calibrated at different plastic work levels.

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